Insurance Companies’ Solvency Management
within the Framework of Logistic Capital Management Theory

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Abstract

Various models of economic growth, addressing the growth trends of country's economy, production, population and other structural objects are focused on the mathematical description of growth rates, taking into account the initial state of the object. One of the solutions to the capital growth rate assessment problem is offered by logistic capital management theory, which is based on the assumption that under the real circumstances the capital usually cannot grow at the same pace for a long time. The article presents an insurance companies’ logistical solvency management model, prepared in accordance with provisions of the logistic capital management theory adjusted in the field of insurance. This model is structurally divided into three main elements: (a) the insurance company's solvency assessment, (2) logistical capital management decisions (3) provisions of the Solvency II project. Logistical insurance companies’ solvency management model shows capital management solutions’ implementation capabilities in the insurance sector, focusing on insurance solvency assessment. This model allows to determine the insurance company’s solvency in respect to the portfolio of an individual client, insurance type and all company, compared the estimated need for insurance benefits (discounted at the current value) with the factual capacity of the insurance company, i.e. available resources to ensure solvency. Also, this model allows planning the insurance activity by making insurance pricing, depending on the projected benefits and fees’ characteristics.

Keywords: insurance, solvency of insurance companies, logistic capital management

JEL Classification: G22, G3

1. Introduction

Insurance in the modern economy and society is one of the most important business and social environment elements, constituting conditions for stability and continuity of businesses and individuals operating independently despite critical environmental effects. The value of insurance services is one of the major insurance issues that determine the complexity of insurance services and insurance services providers’ solvency and liquidity management complexity. Insurance services’ value determination questions are decided by the insurance actuaries, using sophisticated mathematical models, often based on statistical information.

Complex insurance products, especially having the long term operation, such as life insurance, require evaluating a variety of factors, in order to accurately describe the products’ value and the expected earnings and costs of the insurance company during the contract period. Therefore improved methods of the insurance services’ assessment are constantly looked for, allowing more precise definitions of the expected cash flows of services and to identify a reasonable price of the service in line with the service risk for the policyholder and the insurer.

Insurance deposits paid by the insurer are usually specified by exact amount and time at the time of the formation of the agreement on the insurance services delivery, so this part
of the money flows is well predicted and controlled. However, the insurer paid insurance benefits’ cash flow part is of a probabilistic nature, so its prediction is based only on historical data based on statistical analysis. Thus, the insurance services’ value assessment problem is dealt with in order to harmonize the probable and projected cash flows equalizing their values in the course of time. One possible way to assess insurance risk and the resulting price of the service is the logistical capital management theory, adapted to the insurance industry. This theory provides a basis for long-term insurance contracts risk assessment, reflecting the likely cost level of the service, so it is appropriate to examine its application in the insurance sector, the opportunities and potential use.

**Object of the article** - the insurance companies’ solvency management using logistic capital management theory.

**Aim of the article** - to introduce the logistic insurance companies’ solvency management model for the solvency management of insurance companies, using logistic capital management theory.

Preparation of the article questions various authors’ (Girdzijauskas, 2002, 2006; Tsoularis and Wallace, 2002, Bradley 1999, Meyer and Ausubel Yung, 1999; Coppola, Di Lorenzo and Sibilla, 2005, Wilson 2007; Čepinskis and Raškinio, 2005 and so on.) publications, including the insurance company solvency, logistic capital management and the Solvency II project research.

### 2. Logistic Capital Management Concept in the Insurance Sector

Various models of economic growth, addressing the growth trends of country's economy, production, population and other structural objects are focused on the mathematical description of growth rates, taking into account the initial state of the object. Most of the economic growth models can characterize capital gains, but in this case remains an open question how in the best way is reflected the capital gains over time. As pointed out by S. Girdzijauskas and V. Boguslauskas (2005), growth models provide a certain mathematical solution - a mathematical function - which connects a certain primary object size with an actual size of the object at a fixed point in time, and allows further assessment of the likely course of object growth rates.

One of the solutions to the capital growth rate assessment problem is offered by logistic capital management theory, the beginning of which is associated with the works of populations’ researchers, and marked input to this theory of the capital management belongs to S. Girdzijauskas (2002, 2006). Logistical capital management theory is based on the assumption that under real circumstances the capital cannot usually grow at the same pace for a long time. Increasing capital not only accepts the external resistance, but also gets into competition with itself. This is particularly evident in a closed system, which has the required limited resources to support the concrete capital growth. Initial capital growth rate in such a system is gradually decreasing until it finally slows down considerably and stops completely, i.e. it operates under the law of decreasing marginal capital (Girdzijauskas, 2002).

Classic expression of the logistic function describes such a capital growth tendency, which allows to accurately identify the upper and lower capital size limits (Girdzijauskas, 2006):
Here, \( x \) - Capital Accumulation time, 
\( K \) - the population size at some point in time \( t \),
\( t \) - the moment in time,
\( R \) - internal growth rate of population,
\( K_m \) - population saturation level (maximal population size).

The integrated solution of the equation allows to calculate the size of the population at any point in time, given the saturation level, internal growth rate and the current (initial) population size (Bradley, 1999):

\[
K(t) = \frac{K_m}{1 + C \cdot e^{-R t}}
\]  

(3)

\[
C = \frac{1}{K(0)} - \frac{1}{K_m}
\]

Here, \( K \) - the population size at some point in time \( t \),
\( t \) - the moment in time,
\( R \) - internal growth rate of population,
\( K_m \) - population saturation level (maximal population size),
\( C \) - the characterizing index of the initial population size.

A. Tsouaris and J. Wallace (2002) supplemented the proposed P.F. Verhulst (1838) model with a coefficients \( \gamma \) and \( \beta \), \( \alpha \) providing higher maneuvering power to the model:

\[
\frac{dK}{dt} = R e^{\gamma \alpha \left(1 - \frac{K}{K_m}\right)^{\beta}}
\]  

(4)

Here, \( K \) - the population size at some point in time \( t \),
\( t \) - the moment in time.
\( R \) - internal growth rate of population,
\( K_m \) - population saturation level (maximal population size),
\( \gamma \) and \( \beta \), \( \alpha \) - equation parameters, entering the set of positive real numbers.
Girdzijauskas S. (2006), according to the P.F.Verhulst (1838) and his followers’ studies, proposes the mathematically modified (improved) logistic growth formula more in line with the capital growth measurement needs:

\[ K = \frac{K_m \cdot K_0 \cdot (1 + i)^t}{K_m + K_0 \cdot (1 + i)^t - 1} \]  

(5)

Here, \( K_0 \) - the capital size at the initial time, 
\( t \) - the moment in time, 
\( i \) - the capital growth norm (the return), 
\( K_m \) - the maximal capital value.

By introducing to this formula the concept of capital saturation level \( S_0 = \frac{K_0}{K_m} \), the formula is transformed into a more customized form for calculations:

\[ K = \frac{K_0 \cdot (1 + i)^t}{1 + S_0 \cdot (1 + i)^t - 1} \]  

(6)

Here, 
\( K_0 \) - the capital size at the initial time, 
\( t \) - the moment in time, 
\( i \) - the capital growth norm (the return), 
\( S_0 \) - the capital saturation level at the initial time, 
\( K_m \) - the maximal capital value.

S. Girdzijauskas and R. Mackevičius (2009) executed logistic capital growth function’s practical application research showed that the logistic dependence internal profit rate is higher than the classical discounted values, in addition, the increasing saturation factor increases the logistical dependence internal profit rate, which means that increasing the saturation increases the system’s internal return rate.

According to S.Girdzijauskas (2006), A.Tsoularis and J.Wallace (2002) and P.S.Meyer, J.W.Yung and J.H.Ausubel (1999) standpoints of the logistical capital management theory’s adaptation to the insurance fees and benefits management, the insurance premium’s calculation can be expressed through the universal formula suitable for different life insurance situations, regardless of the duration of payment of rents and whether it will be a one-time payment or periodic payment:

\[ \frac{N_A_{x/w}}{\mu} = \sum_{j=1}^{n-x} \frac{\mu^{P_X} \cdot S \cdot S_m}{S_m \cdot (1 + i)^{j-1} - \mu^{P_X} \cdot S \cdot (1 + i)^{j-1} - 1} \]  

(7)

Here, 
\( x \) - age of the insured at the time of the contract, 
\( n \) - the time lag between premium payment and the beginning of receipt of benefits, 
\( W \) - the determined age of the insured at the time of the contract, till which the rent will be paid, taking into account deferred payment, if the rent is fixed, 
\( S \) – size of the insurance sum, 
\( S_m \) - the critical capital resources, 
\( i \) - the capital return (interest rate), 
\( jP_X \) - the likelihood that a person aged \( x \) will survive another \( j \) amount of years, 
\( N_A_{x/w} \) - a one-time insurance premium (current insurance benefits value).
The submitted formula corresponds to A.Tsoularis, J.Wallace (2002) and P.S.Meyer, J.W.Yung and J.H.Ausubel (1999)’s presented preconditions for logistic economic growth assessment involving the logistic curve variation shape, the curve limits pertaining to the variables and the ability to manipulate the function adapting it to the universal changes in capital value variation calculations. It can therefore be suggested that the presented logistic function is an appropriate assessment of the solvency of insurance companies determining the current estimated insurance benefits and evaluating its relationship with the insurance company’s available solvency margin and the expected contributions of policyholders, i.e. solvency margin variation tendencies. From the presented transformed formula it is clear that the insurance premium, showing life insurance coverage net costs (without taking into account the insurance company’s administrative costs) is related by an inverse relationship with the difference between the discounted marginal capital cost and discounted rent payments. The presented formula refers to a one-time insurance premium size, calculated using logistic capital management principles.

In the case of life insurance there often occurs a need to determine a periodic accumulative payments’ size, which discounted to the current value must meet the current value of the future rent. For this S.Girdzijauskas (2006)’s proposed annual bonus formula may be used, highlighting the discount norm’s influence on the rent size and the marginal capital resources’ role in the discounting:

\[
\begin{align*}
P_{x + n} &= \sum_{j=0}^{n-1} \frac{p_{x} \cdot r \cdot s_{m}}{s_{m} \cdot (1 + 0^{i}) - p_{x} \cdot r \cdot (1 + 0^{i} - 1)} \\
\end{align*}
\]

(8)

Here,  
- \( x \) - age of the insured at the time of the contract,
- \( n \) - the time lag between the beginning of the premium payment and starting of receipt of benefits,
- \( P \) - periodic premium size,
- \( S_{m} \) - the critical capital resources,
- \( i \) - the capital return (interest rate),
- \( s_{j} \cdot p_{x} \) - the likelihood that a person aged \( x \) will survive another \( j \) amount of years,
- \( P_{x + n} \) - "current cumulative insurance premiums’ value."

The life insurance base is the existing policyholder’s need to accumulate a certain sum of assets over a specified period of time, after which he could receive the pre-specified (specified or expected) benefits. This concept of life insurance through the prism of financial flows is expressed by M.Coppola, E.Di Lorenzo and M. Sibillo (2005)’s provided formula:

\[
X_{s} = \begin{cases} 
-P_{s}, & \text{if } s < T \\
B_{s}, & \text{if } s \geq T 
\end{cases}
\]

(9)

Here,  
- \( s \) – the moment in time,
- \( T \) - number of years after the conclusion of the contract after which benefits start to be paid,
- \( P_{s} \) - the insurance premiums paid at time \( s \),
- \( B_{s} \) - the insurance benefits paid at time \( s \),
- \( X_{s} \) - financial flow under the insurance contract at the time \( s \).
According to M. Coppola, E. Di Lorenzo and M. Sibillo (2005) mathematically expressed standpoint to the life insurance essence, the periodic premium size according to the above mentioned formulas is calculated comparing rent payments’ (the expected insurance benefits payments) current value with the accumulative insurance premiums’ current cost:

\[ P_{x+n} = nA_{x+n}^{-w} \]  

(10)

Here, \( P_{x+n} \) - current accumulative insurance premiums’ value, 
\( nA_{x+n}^{-w} \) - current value of insurance benefits.

In this case the overall value of life insurance logistic function takes the following mathematical form:

\[
\sum_{j=0}^{n-1} \frac{jP_x \cdot P \cdot S_m}{S_m \cdot (1+i)^j - jP_x \cdot (1+i)^j - 1} = \sum_{j=0}^{n-1} \frac{jP_x \cdot S \cdot S_m}{S_m \cdot (1+i)^j - jP_x \cdot S \cdot (1+i)^j - 1} 
\]  

(11)

Here,  
\( x \) - age of the insured at the time of the contract,  
\( n \) - the time lag between the stating of premium payment and receipt of benefits,  
\( w \) - the age of the insured determined at the time of the contract, until which the rent will be paid,  
\( S \) - size of insurance sum,  
\( P \) - periodic premium size,  
\( S_m \) - the critical capital resources,  
\( i \) - the capital return (interest rate),  
\( jP_x \) - the likelihood that a person aged x will survive another j amount of years,

According to the discussed life insurance value determination, using logistic capital management theory’s principles a logistic insurance solvency management model is formed, as discussed in a later chapter.

3. Insurance Companies’ Solvency Management Model Structure using the Logistic Management Theory

Insurance companies’ logistic solvency management model is based on the insurance companies’ solvency management theoretical aspects and the logistical capital management principles, oriented to the insurance sector. Insurance companies’ logistic solvency management model consists of three main structural components (Fig. 1.)

- Insurance company’s solvency assessment.
- Logistic capital management solutions.
- Project’s Solvency II provisions.
According to the given scheme, the insurance company's solvency assessment is associated with the logistic capital management theory, in the basis of which rates determining the insurance company's solvency are calculated. All calculations and insurance company’s solvency interpretations are associated with the recommendations of the project Solvency II on the insurance company’s business organization and risk management.

Further there are discussed in more detail each of the structural parts of the insurance companies’ logistic solvency management model.

Project’s Solvency II provisions. Solvency II project focuses on insurance companies’ increase in the search options of reliability, with an emphasis on insurance risk assessment and the precise characterization of the risk obligations of the insurers. According to T.C. Wilson (2007), A. Chen (2007) and the Deloitte (2008) publications, there can be excluded the following key provisions of the Solvency II project forming the principles of logistic capital management theory’s application to the insurance company’s solvency assessment:

- **The tariff adequacy to the risk.** Insurance tariffs (insurance fees) have to be proportionate as much as possible to the risks of the particular insured, it is important to accurately identify the risk level of each case, beyond the average estimations.

- **The adequacy of technical postponements.** The insurer must be able to accurately assess the need for technical postponements, depending on the assumed risk of the portfolio. Technical postponements must reflect the overall risk of the different types of insurance and to be sufficient in individual cases and the whole portfolio risk coverage.

- **Risk assessment-based capital requirements.** Insurer's capital must meet the level of the risk assumed, ensuring the insurer's ability to fulfill his obligations to the insured events.

Insurance company’s solvency assessment is based on three elements - the insurance services, insurance fees and risk - interfaces. Solvency assessment relevance arises from the insurance service appearance fact – when a client displays a wish to use an insurance service, form certain provisions of the insurance risk assessment and the determination of insurance fee. According to J.Čepinsko (1999), L.Belinskajos, K. Bagdonavičius and A.Šerniaus (2001), J.Čepinsko and D. Raškinio (2005) and V. Kindurys (2002) publications, it can be argued that, in general, insurance service is characterized by three main criteria:

- **The insured events.** The essence of the insurance service is the need to hedge against certain events that may have adverse financial or other consequences.
Therefore, as stated by M. Guillen, J. Parner and C. Densgsoe and A.M. Perez-Marin (2002) and J.H. Wilson (2005), an insurance service primarily determines exactly what events it covers and what insurance risk such events cause.

- **Insurance protection's duration.** Insurance coverage is usually valid for a specified period of time on the length of which depends the insurance company’s assumed risk level - longer duration of insurance coverage usually results in a higher probability of occurrence of the insured event (or if more of the insured events will occur), which leads to higher insurance service risk.

- **The features of the insured.** The risk of the insured event is due to a large extent to external factors beyond the control of the insured, but it also affects the size and frequency of insured events, therefore individual features as well characterize the insurance service.

The insurance service, depending on its characteristics, describes the insurance risks and forms the insurance fee. Insurance risk conditioning the insurance company’s solvency according to L. Belinskaja and K. Bagdonavičius, A. Šernius (2001), J. Ėpiniskiu, D. Raškinis, (2005), A. Linartas (2003) and M. Peičius (2005), can be divided into two parts:

- **Insured events’ risk,** which is directly linked to insured events’ likelihood and extent assessment. This risk component refers to the expected benefits for the insured events during the insurance protection validity period.

- **Capital market risks** associated with assets held by the insurance company financial management performance. This component of risk affects the determination of the needs for fees for the insurance protection, assessing the capital market characteristics.

- Depending on the benefits and fees for the need of insurance protection and their compatibility, a full insurance fee is formed, which, according to P. Ėpiniskis (1999), includes:

  - **Risk underwriting costs,** which depend on the characteristics of the insured event (extent, probability, nature, etc.) expressed through the insured events’ risk assessment.

  - **Capital accumulation premium,** which describes the insurer's expectations in capital market, in the form of investment accumulating policyholders' fees to cover foreseeable insured events.

  - **Service costs** which have to cover administrative costs suffered by the insurer and to create preconditions for an economically reasonable profit earning.

The insurance fee size for the bigger part is determined by risk underwriting costs and by the capital accumulation premium’s elements, depending on the estimated level of risk and expected capital gains. In this area the logistic capital management theory’s impact on the solvency of insurance companies is displayed.

**Logistic capital management solutions.** Logistic capital management theory is adjusted to the insurance company’s solvency assessment distinguishing two procedures’ subsystems:

- **Capital flows’ compatibility,** where insurance benefits and insurance companies’ assessment and mutual coordination procedures are performed.

- **Logistic capital accumulation calculations,** where the mathematical annuity and insurance fee rate setting procedures are carried out using logistic capital value evaluation formulas.

Capital flows’ compatibility includes the benefits and fees’ demand prognosis, depending on the insurance company’s solvency conditioning factors.
Insurance benefits’ prognosis is formed by insured events’ risk assessment, which, using the logistic functions, is transformed into a financial - a numeric form of the expected payments associated with the occurrence of insured events, assessing their potential magnitude and probability. In turn, the insurance benefits’ prognosis includes capital accumulation premiums and risks underwriting costs’ elements in a full insurance fee composition.

Insurance fee demand is conditioned by the insurance benefits’ forecast and capital market’s expectations, which in the capital market risk form are converted into a payable or receivable supplement to the basic contribution to the insured events coverage.

Table 1. Logistic capital accumulation functions for the insurance companies’ solvency management

<table>
<thead>
<tr>
<th>Title</th>
<th>Function</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross life insurance value’s logistic function</td>
<td>( P_{x+n} = \frac{n}{W} A_{x/n} )</td>
<td>( P_{x+n} ) - current accumulative insurance premiums’ value, ( A_{x/n} ) - current insurance benefits’ value</td>
</tr>
<tr>
<td>Current insurance benefits’ value ( (u A_{x/w}) )</td>
<td>( u A_{x/w} = \sum_{j=m}^{w-x} \frac{p_x \cdot S \cdot S_m}{p_x \cdot S + (S_m - p_x \cdot S) \cdot (1+i)^j} )</td>
<td>( x ) - age of the insured at the time of the contract, ( n ) - the time lag between the beginning of premium payment and receipt of benefits, ( W ) - the age of the insured determined at the time of contract, till which the rent will be paid, taking into account deferred payment, if the rent is fixed ( S ) - size of insurance sum, ( S_m ) - the critical capital resources, ( i ) - the capital return (interest rate) ( p_x ) - the likelihood that a person aged ( x ) will survive another ( j ) amount of years, ( u A_{x/w} ) - a one-time insurance premium (current value of insurance benefits)</td>
</tr>
<tr>
<td>Current accumulative insurance premiums’ value ( (P_{x+n}) )</td>
<td>( P_{x+n} = \sum_{j=m}^{n} \frac{p_x \cdot P \cdot S_m}{p_x \cdot P + (P \cdot (1+i)^j - 1)} )</td>
<td>( x ) - age of the insured at the time of the contract, ( n ) - the time lag between the beginning of premium payment and receipt of benefits, ( P ) - periodic premium size, ( S_m ) - the critical capital resources, ( i ) - the capital return (interest rate) ( p_x ) - the likelihood that a person aged ( x ) will survive another ( j ) amount of years, ( P_{x+n} ) - current value of cumulative insurance premiums</td>
</tr>
</tbody>
</table>
Insurance payments’ forecast and payments’ demand is determined using logistic capital accumulation functions, which are given in Table 1.

In the table the current benefits’ values, current accumulative insurance benefits’values and gross life insurance values logistic function are presented according to S.Girdzijauskas (2006), A.Tsoularis, A.Wallace (2002) and P.S.Meyer, J.W.Yung and J.H.Ausubel (1999), which allow to set the standard demand for the insurance services flows and the target size, linking insurance benefits, characterizing the risk underwriting costs and capital accumulation premiums with insurance fees, which are influenced by the capital market’s risk variables. According to the formulas, the insurance benefits’ forecast fits with an annuity rate that corresponds to the expected need for insurance payments, depending on the insured events’ risk, insurance fees’ demand is associated with the two main market characteristics: capital saturation limit and capital cost rate. Graphically discussed logistic insurance companies’ solvency management model is shown in Fig. 2.

**Figure 2.** Insurance companies’ logistic solvency management model
Insurance companies’ logistic solvency management model shows logistic capital management solution’s implementation capabilities in the insurance sector, focusing on insurance companies’ solvency assessment. This model allows to determine the insurance company’s solvency in respect of the portfolio of an individual client, insurance type and the whole insurance company, comparing the estimated need for insurance benefits (discounted at the current value) to the actual capability of the insurance company, i.e. available resources to ensure solvency. Also, this model allows planning the business of insurance, making insurance service pricing, depending on the forecasted benefits’ and fees’ characteristics.

4. Findings

1. Logistic capital management theory is based on the assumption that under the real circumstances the capital usually cannot grow at the same pace for a long time. Increasing capital not only accepts the external resistance, but also gets in competition with itself, therefore long-lasting capital accumulation has non-linear variation expression, and it is important in insurance, especially life, business where long-term capital flows predominate and where capital saturation limit may have a significant impact on the company's solvency and business continuity capabilities.

2. Logistic capital management theory in the case of life insurance allows you to determine the contract value in current time and its change over time, taking into account the accumulated capital’s available saturation point, which limits the rate of capital growth over time. Applying the same capital accumulation rules to the accumulated and paid amounts of insurance contract, insurance company’s solvency can be associated with the insurance contracts’ portfolio value variation in the long-term perspective.

3. The life insurance premium calculation can be expressed as a universal formula based on the logistic capital management theory, which is suitable for different life insurance situations, regardless of what is the duration of the rents’ payment and on whether it will be a one-time payment or periodic payment. This formula corresponds to the logistic economic growth assessment assumptions involving the logistic curve variation shape, the variables specifying curve limits and the ability to manipulate the function adapting it to the universal capital value variation calculations. For this reason, the formula presented in the article is suitable for various life insurance products and can be used as a basis for calculating the values of life insurance contracts. Insurance companies’ logistic solvency management model structure shows that the insurance company's solvency assessment is related to the logistic capital management theory, on the basis of which executed calculations are determining the insurance company’s solvency rates. All calculations and insurance company’s solvency interpretation is associated with the recommendations of the Solvency II project for the organization of an insurance company’s activity and risk management. Logistic capital management theory is a key element of the model that determines the value of insurance contracts and its dependence on time, which is a direct reflection of the insurance company's solvency indicators.

4. Insurance companies’ logistic solvency management model is structurally divided into three main elements: (1) the insurance company's solvency assessment, where the solvency is related to the insurance contract cash flows, (2) logistical capital management solutions on the basis of which it is determined the value of insurance contracts and its change in the long-term perspective, and (3) the Solvency II project provisions pertaining to the insurance company’s solvency management orientation references. Insurance
companies’ logistic solvency management model shows logistic capital management solutions’ implementation capabilities in the insurance sector, focusing on insurance solvency assessment.

5. The presented model includes the basic life insurance companies’ solvency assessment aspects directly associating solvency with the insurance contracts’ current and future values’ comparison and evaluation of cash flows. This makes it possible to predict the changes in the solvency of insurance companies, depending on the capital market and insured events’ risks also taking into account the accumulated capital’s characteristic to change within the limits of the logistic function.

References: